Symmetries Then and Now

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Global symmetries are useful

- If unbroken
  - Multiplets
  - Selection rules
- If broken
  - Goldstone bosons for continuous
  - Domain walls for discrete
- Landau’s classification of phases
- More sophisticated: ‘t Hooft anomalies
- **But no global symmetries in gravity**
Gauge symmetry is deep

• Largest symmetry (a group for each point in spacetime)
• Useful in making the theory manifestly Lorentz invariant, unitary and local (and hence causal)
• Appears in
  • Maxwell theory, the Standard Model
  • General Relativity
  • Many condensed matter systems
  • Deep mathematics (fiber bundles)
But

• Because of Gauss law the Hilbert space is gauge invariant. (More precisely, it is invariant under small gauge transformation; large gauge transformations are central.)

• Hence: **gauge symmetry is not a symmetry.**
  • It does not act on anything.

• A better phrase is **gauge redundancy.**
Gauge symmetry can appear trivial

• Start with an arbitrary system and consider some transformation, say a $U(1)$ phase rotation on some fields. It is not a symmetry.

• Introduce a Stueckelberg field $\phi(x)$, which transforms under the $U(1)$ by a shift.

• Next, multiply every non-invariant term by an appropriate phase $e^{i\phi(x)}$, such that the system has a local $U(1)$ gauge symmetry.
  • $e^{i\phi(x)}$ is a nowhere vanishing section, so only trivial bundles – no monopole flux.

• Clearly, this is not a fundamental symmetry.
Gauge symmetry is always unbroken

• Not a symmetry and hence cannot break
• For spontaneous symmetry breaking we need an infinite number of degrees of freedom transforming under the symmetry. Not here.
• Hence, there is no massless Goldstone boson.
• For weakly coupled systems (like Landau-Ginsburg theory of superconductivity, or the weak interactions) the language of spontaneous symmetry breaking is perfectly appropriate and extremely useful [Stueckelberg, Anderson, Brout, Englert, Higgs, ...].
Global symmetries can emerge as accidental symmetries at long distance. Then they are approximate.

Exact gauge symmetries can be emergent.
Examples of emergent gauge symmetry

• The example with the added field $\phi(x)$ above.
• Some $\sigma$-models can be described using gauge fields (e.g. the $CP^N$ $\sigma$-model) and then they become dynamical.
  – This is common in condensed matter physics.
• Simple dualities
  – In $3d$ a compact scalar is dual to Maxwell theory, whose gauge symmetry is emergent.
  – In $4d$ Maxwell theory is dual to a magnetic Maxwell theory.
Duality in interacting field theories

\[ N = 4 \] supersymmetry

• This is a scale invariant theory characterized by a
gauge group \( G \) and a complex coupling constant
\[ \tau = \frac{\theta}{2\pi} + \frac{4\pi}{g^2} i \] for each factor in \( G \).

• For simply laced \( G \) the theory with \( \tau \) is the same as
with \( \tau + 1 \) (shift \( \theta \) by \( 2\pi \)) and the same as with \(-1/\tau\)
generating \( SL(2, \mathbb{Z}) \)...
Duality in interacting field theories
\( N = 4 \) supersymmetry

• The duality is an exact equivalence of theories.
  • Same spectrum of states
  • Same spectrum of operators
  • Same correlation functions
• \( \tau \to -1/\tau \) maps strong to week coupling.
• More technical:
  • This ignores certain global issues.
  • Some modifications for non-simply laced \( G \).
Duality in interacting field theories

\[ N = 4 \] supersymmetry

- The gauge symmetry of the dual description is emergent!
- Which of the two gauge symmetries is fundamental?
- Which set of gluons is elementary?
- More likely, neither gauge symmetry is fundamental.
Interacting gauge theories

Start at short distance with a gauge group $G$. Depending on the details we end up at long distance with:

- IR freedom – a free theory based on $G$ (same theory)
- A nontrivial fixed point. Interacting conformal field theory – no notion of particles.
- An approximately free (IR free) theory of bound states
- An empty theory – gap (possible topological order)

All these options are realized in QCD for various numbers of flavors. (The approximately free theory is a theory of pions.)
Duality in interacting field theories

$N = 1$ supersymmetry

Here there is also a dual description based on another gauge theory with gauge group $\tilde{G}$ (magnetic theory).

• When the original theory (electric) is IR free the dual theory is strongly coupled.

• When the electric theory flows to a non-trivial fixed point so is the magnetic theory. The two theories are in the same universality class...
Duality in interacting field theories

$N = 1$ supersymmetry

Electric theory $G$

Magnetic theory $\tilde{G}$

Non-trivial IR fixed point
Duality in interacting field theories

$N = 1$ supersymmetry

A third option:

- Electric theory
  - Based on $G$

  \[ \downarrow \]

- Approximately free theory (IR free)
  - Based on $\tilde{G}$
Duality in interacting field theories
\[ N = 1 \] supersymmetry

In the **UV** an asymptotically free theory based on \( G \)
In the **IR** an IR free theory based on \( \tilde{G} \)

At low energies QCD has pions. This theory has a non-Abelian gauge theory.

- The gauge fields of \( \tilde{G} \) are composite.
- Their gauge symmetry is emergent.
- There is no ambiguity in the IR gauge symmetry – approximately free massless gauge fields.
Duality in interacting field theories

$N = 1$ supersymmetry

In all these cases

• As the original electric theory becomes more strongly coupled, the magnetic theory becomes more weakly coupled.

• When the electric theory confines the magnetic theory exhibits spontaneous gauge symmetry breaking (meaningful because it is weakly coupled).

• Clear physical demonstration of dynamical properties of gauge theories. In particular, emergent gauge fields.
Many more examples of emergent gauge symmetries

• Many known examples based on different
  – gauge groups and matter representations
  – spacetime dimensions
  – amount of supersymmetry
• They exhibit rich physical phenomena.
• They lead to interesting mathematics (many applications).
• **Duality and emergent gauge symmetry are ubiquitous.**
Emergent general covariance and emergent spacetime

- So far we discussed duality between two field theories
- String-string duality
  - T-duality
  - S-duality
  - U-duality
- String-fields duality
  - Matrix models for low dimensional string theories
  - BFSS M(atrix) model
  - AdS/CFT
  - More generally gauge-gravity duality
Generalized Global Symmetries


• View QFT as a collection of ops and their correlation functions
  – Include local and extended observables: lines, surfaces, etc.

• The gauged version of these symmetries
  – In physics Kalb-Ramond and later Villain
  – In mathematics Cheeger-Simons theory

• As with ordinary symmetries:
  – Should study the global symmetry first and then gauge it.
  – Gauge symmetry is not intrinsic
  – Global symmetry is unambiguous
Generalized Global Symmetries

• Ordinary global symmetries
  – Charge: operator associated with co-dimension one manifolds, e.g. whole of space
  – The charged operators are point-like
  – The charged states are particles (0-branes)
• Generalization: $q$-form global symmetries
  – Charge: operator of co-dimension $q + 1$-manifold
  – The charged operators are of dimension $q$, e.g. Wilson and ‘t Hooft lines
  – The charged states are $q$-dimensional ($q$-branes), e.g. strings
$q$-form global symmetries

As with ordinary symmetries:

• Selection rules on amplitudes
• Couple to a background classical gauge field (twisted boundary conditions)
  – Interpret 't Hooft twisted boundary conditions as an observable in the untwisted theory
• Gauging the symmetry by summing over twisted sectors (like orbifolds)
  – New parameters in gauge theories – discrete $\theta$-parameters (like discrete torsion)
• Dual theories often have different gauge symmetries. But the global symmetries must be the same
  – non-trivial tests of duality including non-BPS operators
$q$-form global symmetries

As with ordinary symmetries:

- The symmetry could be spontaneously broken
  - Continuous: the photon is a Goldstone boson
  - Discrete: a topological theory in the IR. Long range topological order

- Anomalies and anomaly inflow on boundaries or defects
  - ‘t Hooft matching – Symmetry Protected Topological phases

- Characterize phases – unified description, extending Landau’s
  - Confinement = one-form global symmetry unbroken
    - Strings, loops with area law
  - Higgs or Coulomb = one-form global symmetry broken
  - Various other phases (mixed, oblique, partial breaking)
Conclusions

• Gauge symmetries are not fundamental. They can emerge in the IR without being present in the UV.
  – It is often convenient to use them to make the description manifestly Lorentz invariant, unitary and local.
  – But there can be different such (dual) descriptions.

• There is a lot more to learn about gauge and global symmetries
Happy 40th Anniversary